# Scattering at the Junction of a Rectangular Waveguide and a Larger Circular Waveguide

Robert H. MacPhie, *Fellow, IEEE*, and Ke-Li Wu, *Member, IEEE*

***Abstract*—A full wave, formally exact solution is obtained for the problem of scattering at the junction of a rectangular waveguide and a larger circular waveguide. The general case of an arbitrary offset of the waveguide axes is considered. $E$-field mode matching over the rectangular aperture of the smaller guide is facilitated by a transformation of the circular cylindrical Bessel–Fourier modal fields of the circular guide into a finite series of exponential plane wave functions in rectangular coordinates. This permits an analytical finite series solution for each of the elements of the $E$-field mode matching matrix $[M]$ from which the scattering matrix $[S]$ of the junction is easily obtained. Numerical evaluation of the $S$-parameters for the dominant $TE_{10}$ (rectangular) and $TE_{11}$ (circular) modes in the cases of junctions with no offset and with offset is presented. Moreover, the practical case of a circular cavity resonator with smaller input and output rectangular guides is considered and excellent agreement is found between the calculated and measured $S$-parameters.**

## I. Introduction

IN A RECENT paper [1], a rigorous full wave solution was obtained for the problem of electromagnetic scattering at the junction of a circular waveguide and a larger rectangular waveguide. In the present paper we consider the complementary problem of the junction of a rectangular guide with a larger circular guide. The transverse Bessel–Fourier modal $E$-fields of the circular guide are expanded in terms of a series of exponential plane wave functions [2] in cartesian coordinates for $E$-field mode matching at the junction's rectangular aperture.

This problem has been very recently treated by Keller and Arndt [3] but no specific details of the coupling integrals [3, (5)] were provided by the authors. In Section II we deduce expressions for the elements of the $E$-field mode matching matrix $[M]$ for the junction analogous to those given in [1]. Moreover, in the Appendix is presented the derivation of the plane wave series expansion of $J_q(h\rho)\exp(jq\phi)$, the Bessel–Fourier modal eigenfunction in the circular waveguide. Section III indicates how the junction's scattering matrix $[S]$ is obtained from $[M]$ and the modal admittance matrices $[Y_1]$ and $[Y_2]$ of the two guides.

Section IV presents numerical results for junctions with no offset and with offset. Moreover, the practical and interesting case of a circular waveguide resonator connected to smaller rectangular guides is considered and the theoretical results are compared with experimental measurements with excellent agreement obtained in all cases.

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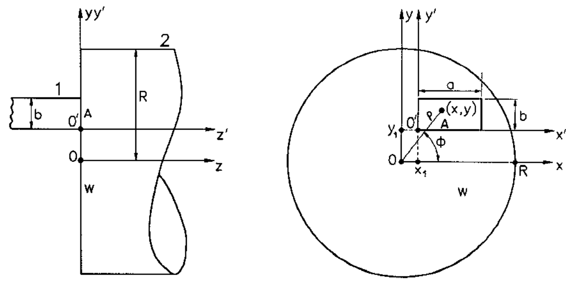


Fig. 1. The junction of a rectangular waveguide with a larger offset circular waveguide.

## II. Electric Field Mode Matching at the Junction

Fig. 1 indicates the geometry of the junction of a rectangular waveguide with a larger circular waveguide. The rectangular guide, with cartesian coordinates $(x', y', z')$, is transversely offset from the circular guide with cartesian coordinates $(x, y, z)$ such that $x' = x - x_1$, $y' = y - y_1$, $z' = z$.

In the rectangular guide, just to the left of the junction ($z' = 0_-$), the tangential electric field can be expanded as a superposition of $TE$ ($h$-type) and $TM$ ($e$-type) modal fields

$$\vec{E}_1(x', y') = \sum_{mn}[A_{1,mn}\vec{e}^{(h)}_{1,mn}(x', y') + B_{1,mn}\vec{e}^{(e)}_{1,mn}(x', y')]. \tag{1}$$

In the larger circular guide, just to the right ($z = 0_+$) of the junction, the tangential electric field can also be expanded as a sum of modes

$$\vec{E}_2(\rho, \phi) = \sum_{qr}[A_{2C,qr}\vec{e}^{(h)}_{2C,qr}(\rho, \phi) + A_{2S,qr}\vec{e}^{(h)}_{2S,qr}(\rho, \phi) + B_{2C,qr}\vec{e}^{(e)}_{2C,qr}(\rho, \phi) + B_{2S,qr}\vec{e}^{(e)}_{2S,qr}(\rho, \phi)]. \tag{2}$$

The various waveguide modal fields in (1) and (2) are specified in Collin [4, pp. 189 and 197], respectively. By means of the equation (see the Appendix for its derivation)

$$J_q(h\rho)e^{jq\phi} \simeq \frac{j^q}{N}\sum_{l=0}^{N-1} e^{j\frac{2lq\pi}{N}} e^{-jh(C_l x + S_l y)} \tag{3}$$

where $x = \rho\cos\phi$, $y = \rho\sin\phi$, $C_l = \cos\left(\frac{l2\pi}{N}\right)$, $S_l = \sin\left(\frac{l2\pi}{N}\right)$ and $N - 1 > h\rho + N_0$ with $N_0$ a small integer, the cylindrical waveguide modal functions can be transformed into cartesian coordinates

$$\vec{e}^{(h)}_{2\binom{C}{S},\, qr}(x,\, y) = \frac{h'_{qr}}{N} j^{q+1} \sum_{l=0}^{N-1} \binom{C_{lq}}{S_{lq}} \cdot(\hat{x}S_l - \hat{y}C_l)e^{-jh'_{qr}(C_lx+S_ly)} \quad (4)$$

$$\vec{e}^{(e)}_{2\binom{C}{S},\, qr}(x,\, y) = \frac{h_{qr}}{N} j^{q+1} \sum_{l=0}^{N-1} \binom{C_{lq}}{-S_{lq}} \cdot(\hat{x}C_l + \hat{y}S_l)e^{-jh_{qr}(C_lx+S_ly)}. \quad (5)$$

Then we can use the relations $x = x' + x_1$, and $y = y' + y_1$ to express the circular waveguide mode fields in guide 2 in terms of the Cartesian coordinates of the rectangular guide 1

$$\vec{e}^{(h)}_{2\binom{C}{S},\, qr}(x',\, y') = \frac{h'_{qr}}{N} j^{q+1} \sum_{l=0}^{N-1} \binom{C_{lq}}{S_{lq}} e^{-jh'_{qr}(C_lx_1+S_ly_1)} \cdot(\hat{x}S_l - \hat{y}C_l)e^{-jh'_{qr}(C_lx'+S_ly')} \quad (6)$$

$$\vec{e}^{(e)}_{2\binom{C}{S},\, qr}(x',\, y') = \frac{h_{qr}}{N} j^{q+1} \sum_{l=0}^{N-1} \binom{C_{lq}}{-S_{lq}} e^{-jh_{qr}(C_lx_1+S_ly_1)} \cdot(\hat{x}C_l + \hat{y}S_l)e^{-jh_{qr}(C_lx'+S_ly')}. \quad (7)$$

In (4),...,(7), $h'_{qr}R$ and $h_{qr}R$ are respectively the $r$th roots of $J'_q(x)$ and $J_q(x)$; moreover $C_{lq} = \cos\left(lq\frac{2\pi}{N}\right)$ and $S_{lq} = \sin\left(lq\frac{2\pi}{N}\right)$.

We use (6) and (7), together with the traditional Cartesian coordinate expressions for the rectangular modal fields [4, p. 189], in the $E$-field mode matching matrix integrals

$$\begin{Bmatrix} H^C_{qr,\, mn} \\ K^C_{qr,\, mn} \end{Bmatrix} = \frac{1}{N^{(h)2}_{2,\, qr}} \int_o^a \int_o^b \vec{e}^{(h)}_{2C,\, qr}(x',\, y') \cdot \begin{Bmatrix} \vec{e}^{(h)}_{1,\, mn}(x',\, y') \\ \vec{e}^{(e)}_{1,\, mn}(x',\, y') \end{Bmatrix} dx'\, dy' \quad (8)$$

$$\begin{Bmatrix} Q^C_{qr,\, mn} \\ E^C_{qr,\, mn} \end{Bmatrix} = \frac{1}{N^{(e)2}_{2,\, qr}} \int_o^a \int_o^b \vec{e}^{(e)}_{2C,\, qr}(x',\, y') \cdot \begin{Bmatrix} \vec{e}^{(h)}_{1,\, mn}(x',\, y') \\ \vec{e}^{(e)}_{1,\, mn}(x',\, y') \end{Bmatrix} dx'\, dy' \quad (9)$$

where $N^{(h)}_{2,\, qr}$ and $N^{(e)}_{2,\, qr}$ are the norms of the circular guide's $qr$th $h$ and $e$ modes respectively. A similar set of expressions for the sine-type coefficients is obtained by replacing $\vec{e}^{(h)}_{2C,\, qr}$ and $\vec{e}^{(e)}_{2C,\, qr}$ by $\vec{e}^{(h)}_{2S,\, qr}$ and $\vec{e}^{(e)}_{2S,\, qr}$ in (8) and (9), respectively. In total, the $E$-field mode matching matrix equation is as follows

$$\underline{C}_2 = \begin{bmatrix} \underline{A}^C_2 \\ \underline{A}^S_2 \\ \underline{B}^C_2 \\ \underline{B}^S_2 \end{bmatrix} = \begin{bmatrix} [H^C] & [K^C] \\ [H^S] & [K^S] \\ [Q^C] & [E^C] \\ [Q^S] & [E^S] \end{bmatrix} \begin{bmatrix} \underline{A}_1 \\ \underline{B}_1 \end{bmatrix} = [M]\underline{C}_1 \quad (10)$$

where the $mn$th element of the column vector $\underline{A}_1$ is $A_{1mn}$, etc.

It is straightforward to show, using (4) in (8) that

$$H^c_{qr,\, mn} = \frac{j^{q+1}}{N^{(h)2}_{2,\, qr}} \frac{\pi}{N} \sum_{l=0}^{N} C_{lq} e^{-j(C_lx'_{1,\, qr}+S_ly'_{1,\, qr})} \cdot [S_l n a'_{qr} I^C_m(C_l a'_{qr}) I^S_n(S_l b'_{qr}) + C_l m b'_{qr} I^S_m(C_l a'_{qr}) I^C_n(S_l b'_{qr})] \quad (11)$$

where $a'_{qr} = h'_{qr}a$, $b'_{qr} = h'_{qr}b$, $x'_{1,\, qr} = h'_{qr}x_1$ $y'_{l,\, qr} = h'_{qr}y_1$ and

$$I^C_m(\beta d) = \frac{1}{d}\int_o^d e^{-j\beta z} \cos\left(\frac{m\pi}{d}z\right) dz = j\beta d \frac{(-1)^m e^{-j\beta d} - 1}{(\beta d)^2 - (m\pi)^2} \quad (12)$$

$$I^S_m(\beta d) = \frac{1}{d}\int_o^d e^{-j\beta z} \sin\left(\frac{m\pi}{d}z\right) dz = \frac{m\pi}{j\beta d} I^C_m(\beta d). \quad (13)$$

One obtains $H^S_{qr,\, mn}$ by replacing $C_{lq}$ with $S_{lq}$ in (11). Therefore, if

$$L_{qr,\, mn,\, l} = \frac{1}{N} e^{-j(C_lx'_{1,\, qr}+S_ly'_{1,\, qr})} [S_l n a'_{qr} I^C_m(C_l a'_{qr}) I^S_n(S_l b'_{qr}) + C_l m b'_{qr} I^S_m(C_l a'_{qr}) I^C_n(S_l b'_{qr})] \quad (14)$$

then

$$\begin{Bmatrix} H^C_{qr,\, mn} \\ H^S_{qr,\, mn} \end{Bmatrix} = \frac{j^{q+1}\pi}{N^{(h)2}_{2,\, qr}} \sum_{l=0}^{N-1} \begin{Bmatrix} C_{lq} \\ S_{lq} \end{Bmatrix} L_{qr,\, mn,\, l}. \quad (15)$$

In the cases of rectangular-to-rectangular waveguide junctions [6] and of rectangular to smaller circular waveguide junctions [1] the elements of the $[K]$ matrix were identically zero. This is also true in the present case, for both $[K^C]$ and $[K^S]$.

However, for the $[Q]$ and $[E]$ matrices we obtain

$$\begin{Bmatrix} Q^C_{qr,\, mn} \\ Q^S_{qr,\, mn} \end{Bmatrix} = \frac{j^{q+1}\pi}{N^{(e)2}_{2,\, qr}} \sum_{l=0}^{N-1} \begin{Bmatrix} C_{lq} \\ -S_{lq} \end{Bmatrix} M_{qr,\, mn,\, l} \quad (16)$$

where

$$M_{qr,\, mn,\, l} = \frac{1}{N} e^{-j(C_lx_{1,\, qr}+S_ly_{1,\, qr})} \cdot [C_l n a_{qr} I^C_m(C_l a_{qr}) I^S_n(S_l b_{qr}) - S_l m b_{qr} I^S_m(C_l a_{qr}) I^C_n(S_l b_{qr})] \quad (17)$$

and $a_{qr} = h_{qr}a$, etc.,

$$\begin{Bmatrix} E^C_{qr,\, mn} \\ E^S_{qr,\, mn} \end{Bmatrix} = \frac{j^{q+1}\pi}{N^{(e)2}_{2,\, qr}} \sum_{l=0}^{N-1} \begin{Bmatrix} C_{lq} \\ S_{lq} \end{Bmatrix} N_{qr,\, mn,\, l} \quad (18)$$

in which

$$N_{qr,\, mn,\, l} = \frac{1}{N} e^{-j(C_lx_{1,\, qr}+S_ly_{1,\, qr})} \cdot [C_l m b_{qr} I^C_m(C_l a_{qr}) I^S_n(S_l b_{qr}) + S_l n a_{qr} I^C_m(C_l a_{qr}) (I^S_n(S_l b_{qr})]. \quad (19)$$

TABLE I
CONVERGENCE OF THE SCATTERING PARAMETERS FOR THE $TE_{10}$ AND $TE_{11}$ MODES FOR CENTERED GUIDES WITH $a = R = 2b = 0.75''$ AT $f_0 = 9.0$ GHz

| NUMBER OF MODES | | | | $S_{11}$ | $S_{21}$ |
|---|---|---|---|---|---|
| GUIDE 1 | | GUIDE 2 | | | |
| TE | TM | TE | TM | | |
| 4 | 2 | 28 | 14 | -.1272 + j.6780 | -.5703 - j.4459 |
| 8 | 4 | 56 | 28 | -.1498 + j.6784 | -.5639 - j.4492 |
| 14 | 7 | 98 | 49 | -.1520 + j.6805 | -.5571 - j.4510 |
| - | - | - | - | | |

## III. THE SCATTERING MATRIX OF THE JUNCTION

As described in [1], the scattering matrix $[S]$ of the rectangular-circular waveguide junction, which in our present case is such that

$$\underline{C}^- = \begin{bmatrix} \underline{C}_1^- \\ \underline{C}_2^- \end{bmatrix} = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{bmatrix} \underline{C}_1^+ \\ \underline{C}_2^+ \end{bmatrix} = [S]\underline{C}^+ \quad (20)$$

can be obtained in terms of the $E$-field mode matching matrix $[M]$ and the diagonal modal admittance matrices $[Y_i]$ ($i = 1$ and 2) of the two guides (see [1, (31),$\cdots$,(33)]. The various submatrices are as follows

$$[S_{11}] = ([Y_1] + [Y_{L1}])^{-1}([Y_1] - [Y_{L1}]),$$
$$[S_{12}] = 2([Y_1] + [Y_{L1}])^{-1}[M]^T[Y_1] \quad (21)$$

$$[S_{21}] = [M]([S_{11}] + [I]), \qquad [S_{22}] = [M][S_{12}] - [I] \quad (22)$$

where

$$[Y_{L1}] = [M]^T[Y_2][M] \quad (23)$$

is the "load" admittance matrix of guide 2 as "seen" by guide 1, and $[I]$ is the identity matrix. For the cascaded connection of two junctions in series such as a circular cavity fed at each end by smaller rectangular guides, we can use the generalized scattering matrix technique [7, pp. 207–217] to obtain the overall scattering matrix of the structure.

## IV. NUMERICAL RESULTS

To begin, we investigate the convergence of the $S$-parameters $S_{11}$ and $S_{21}$ as functions of the number of modes used. A centered junction (no offset) is considered with a rectangular $WR75$ waveguide ($a = 0.75'' = 2b$) feeding a larger circular guide of radius $a$. The ratio of cross-sectional areas is therefore $\pi a^2/(a^2/2) = 2\pi < 7$; to avoid the relative convergence problem [7], if $N$ rectangular modes ( $TE$ or $TM$) are considered, then in the circular guide we would use $7N$ modes ($TE$ or $TM$). Moreover, since $TE$ modes are dominant, we select twice as many $TE$ modes in each guide. Table I indicates that the scattering parameters for the propagating modes converge to three-figure accuracy when $14TE$ and $7TM$ modes are assumed in the smaller rectangular guide.

In Fig. 2 the behavior of $|S_{11}|$ and $|S_{21}|$, for the same junction as considered in Table I, is given for the frequency range from 8.0 GHz to 15.0 GHz. Fig. 3 presents the case

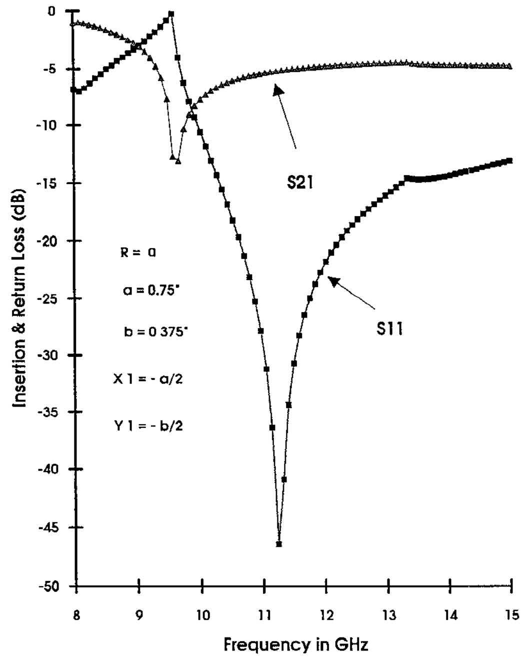


Fig. 2. $S$-parameters $S_{11}$ and $S_{21}$ in dB of the rectangular-to-circular waveguide junction for no offset. The rectangular guide is WR75 with $a = 2b$ =0.75" and the circular guide radius is $R = a$.

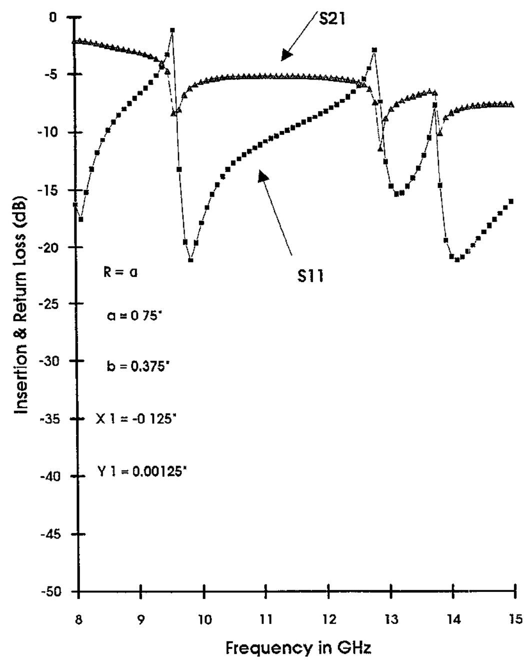


Fig. 3. $S$-parameters in dB of the rectangular-to-circular waveguide junction with offset. The rectangular guide is WR75 with $a = 0.75''$ and the circular guide radius is $R = a$.

of offset waveguides with $x_1$ =0.125", $y_1$ =0.00125". It should be mentioned that the $E$-field mode functions have been normalized by $(Z_{mn}^{(\mu)})^{1/2}$ where $\mu = e, h$, in order to obtain symmetric $S$-parameters ($S_{21} = S_{12}$); $Z_{mn}^{(\mu)}$ is the $mn$th $\mu$-type modal wave impedance.

### A. Experimental Verification

A circular waveguide cavity resonator, fed with no offsets at each end by smaller rectangular waveguides [see Fig.

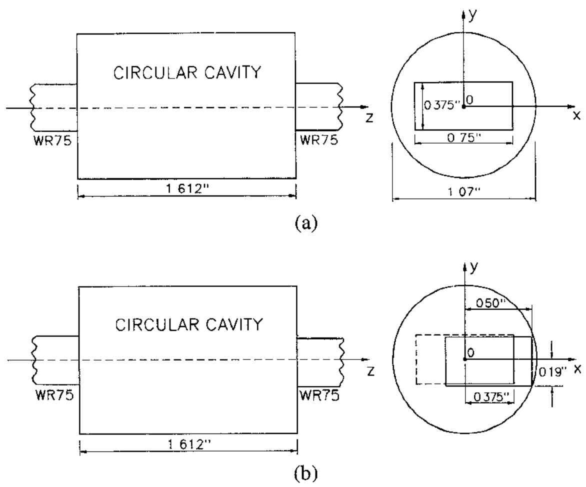


Fig. 4. A circular cylindrical cavity resonator fed by WR75 rectangular waveguides; (a) no offset of the rectangular guides, (b) one of the rectangular guides with offset.

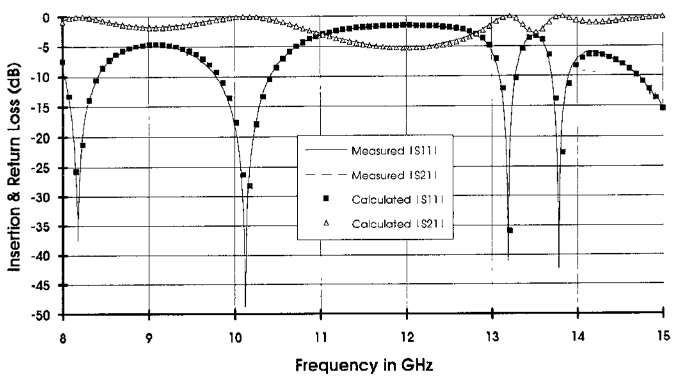


Fig. 5. The calculated and measured $S$-parameters of the circular cylindrical cavity shown in Fig. 4(a).

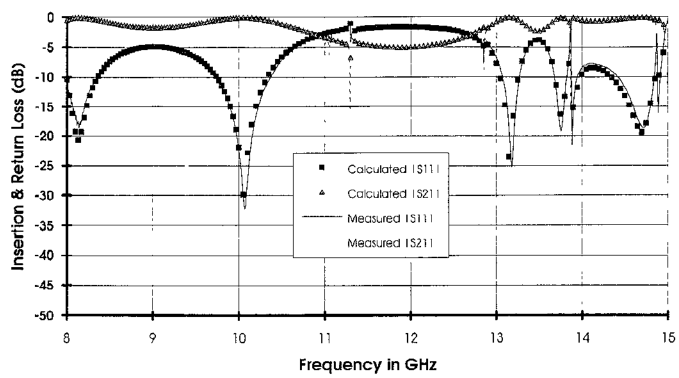


Fig. 6. The calculated and measured $S$-parameters of the circular cylindrical cavity shown in Fig. 4(b).

4(a)] was considered. In Fig. 5 the measured $S$-parameters given by solid/dashed lines are compared with the discrete theoretical values over the 8.0 GHz to 15.0 GHz frequency range. Agreement is excellent.

A second circular guide cavity, with one of the rectangular guides offset, was also considered and the theoretical results are compared to the measured. The geometry is shown in Fig. 4(b) and the experimental and theoretical results are indicated in Fig. 6 where again there is excellent agreement.

## V. Conclusion

This paper has provided a rigorous full wave modal solution to the problem of scattering at a rectangular-to-circular waveguide junction when the circular waveguide is larger in cross section. The new and very useful plane wave series representation of the Bessel–Fourier eigenfunctions which represents the modes in the circular guide leads to analytical series expressions for the elements of the $E$-field mode matching matrix $[M]$ from which the scattering matrix of the junction can easily be deduced. This totally eliminates any numerical integration which, heretofore, has been used in such problems. The usefulness and accuracy of the technique has been demonstrated by the excellent agreement between calculated and measured $S$ parameters of a circular cavity resonator fed at each end by WR 75 rectangular waveguide. Obviously the general scattering solution of this junction will be widely used in the characterization of various waveguide systems such as the slot iris coupling in circular waveguide filters, certain transformers in antenna feed networks and channel filters for space applications.

## Appendix
### Plane Wave Series Expansion of $J_q(h\rho)e^{jq\phi}$

From Stratton [8, pp. 371–372] one obtains the Bessel–Fourier series expansion of a plane wave function

$$e^{-j\vec{k}_i\cdot\vec{r}} = e^{-jk\rho\sin\theta_i(\cos\phi-\phi_i)}e^{-jk\cos\theta_i z} = \sum_{q=-\infty}^{\infty} J_q(k\sin\theta_i\rho)e^{jq\phi}j^{-q}e^{-jq\phi_i}e^{-jk\cos\theta_i z}. \tag{24}$$

Letting $z = 0$ and $k\sin\theta_i = h$, we simplify (24) to

$$\sum_{q=-\infty}^{\infty} j^{-q}e^{-jq\phi_i}J_q(h\rho)e^{jq\phi} = e^{-jh\rho\cos(\phi-\phi_i)}. \tag{25}$$

But if $|q|$ exceeds $h\rho$ the amplitudes of $J_q(h\rho)$ and $J_{-q}(h\rho)$ quickly become negligible [5, p. 359]. Therefore, to a very good approximation

$$\sum_{q=-\overline{N}}^{\overline{N}} j^{-q}e^{-jq\phi_i}J_q(h\rho)e^{jq\phi} = e^{-jh\rho\cos(\phi-\phi_i)} = e^{-jh(\cos\phi_i x+\sin\phi_i y)} \tag{26}$$

where $h\rho < \overline{N} - N_o$, ($N_o$ is a small integer).

We now consider $2\overline{N}+1$ plane waves with directions of incidence $\phi_{il}$ given by

$$\phi_{il} = l\,\Delta\phi = l\frac{2\pi}{N}, \qquad l = 0, 1, \ldots 2\overline{N}, \quad (N = 2\overline{N}+1). \tag{27}$$

Then, after multiplication by $N^{-1}$, (26) becomes

$$\frac{1}{N}\sum_{r=0}^{N-1} e^{-jlr\frac{2\pi}{N}}B_r = W_l \tag{28}$$

TABLE II
CONVERGENCE OF THE SERIES EXPANSION OF $J_n(h\rho)\exp jn\phi$ IN CARTESIAN COORDINATES FOR $n = 8$, $h\rho = 19.670$ , $\phi = 38°$

| N | $J_8(h\rho)\,e^{j8\phi} \cong \frac{j^8}{N}\sum_{l=0}^{N-1} j^{l\frac{16\pi}{N}}\, e^{jh(C_l x + S_l y)}$ <br> $h = p'_{85}/R,\ \rho = 0.8R\ :\ h\rho = 19.670,\ \phi = 38°$ |
|---|---|
| 19 | - 0.0111659 + j 0.0162155 |
| 21 | - 0.0110807 + j 0.0164307 |
| 23 | - 0.0110803 + j 0.0164273 |
| - | |
| $J_8(19.670)\,e^{j304°} = -\ 0.0110803 + j\,0.0164273$ | |

where

$$B_r = j^{-(r-\overline{N})} J_{r-\overline{N}}(h\rho) e^{j(r-\overline{N})\phi},$$
$$W_l = \frac{1}{N} e^{-jl\frac{N-1}{N}\pi} e^{-jh(C_l x + S_l y)}. \quad (29)$$

Equation (28) is the discrete Fourier transform [9, pp. 358, 359] of the sequence $\{B_0, B_1 \ldots B_{N-1}\}$. Conversely

$$B_r = \sum_{l=0}^{N-1} e^{jlr\frac{2\pi}{N}} W_l. \quad (30)$$

Finally, if $q = r - \overline{N}$ we can use (29) and (30) to obtain

$$J_q(h\rho)e^{jq\phi} = \frac{j^q}{N}\sum_{l=0}^{N-1} e^{jl\frac{2q\pi}{N}} e^{-jh(C_l x + S_l y)}. \quad (31)$$

Table II provides some numerical results showing the convergence of this $N$-term series. The case considered is for a typical higher order $TE$ mode with associated Bessel–Fourier eigenfunction

$$J_8\left(\frac{p'_{85}}{R}\rho\right)e^{j8\phi} = J_8\left(24.587\frac{\rho}{R}\right)e^{j8\phi}. \quad (32)$$

A typical point in the guide is chosen $(\rho, \phi) = (.8R, 38°)$; consequently $24.587\frac{\rho}{R}$ =19.670 and $8\phi = 304°$. In Table I we see that when the number $N = 2\overline{N} + 1$ of terms in the series exceeds the argument of the Bessel function (19.670) the series rapidly converges to the true solution obtained by the product of the conventional series expansions for $J_8(19.670)$ and for $e^{j\,304°}$.

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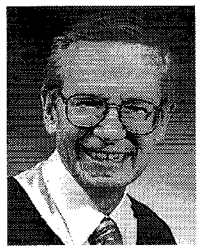


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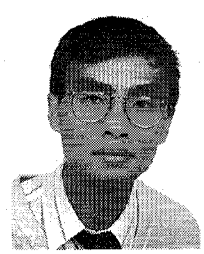


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